

Simultaneous continuous measurement of photon counting and homodyne detection on a free photon field: dynamics and measurement back action

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Abstract

We analyze a simultaneous continuous measurement of photon counting and homodyne detection. The stochastic master equation or stochastic Schrödinger equation describing the measurement process includes both jump-type and diffusive-type stochastic increments. Analytic expressions of the wave function conditioned on homodyne and photocount records are obtained, yielding the probability density distributions and generating functions of the measurement records. The obtained results are applied to typical initial conditions — coherent, number, thermal, and squeezed states. Monte Carlo simulations of the measurement processes are also presented.

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I. INTRODUCTION

Since the birth of quantum theory, the problem of quantum measurement has been among the most thought-provoking problems in physics. In the early stage of quantum theory, the postulate of projective measurements, also known as von Neumann measurements, was put forth by von Neumann [1]. The von Neumann's postulate, however, is not applicable to most experimental setups in a straightforward manner. A more general theory of quantum measurement which can describe non-projective measurements such as photon-counting measurement [2] was formulated much later [3], and it was clarified that the theory of measurement is closely related to the ideas of open quantum systems and quantum operations [4, 5].

Many open quantum systems are known to obey a special form of master equation, called the Lindblad master equation. In a seminal paper [6], Lindblad showed that this master equation can be derived from the mathematical requirement that the quantum operational map Φ_t take the semigroup form $\exp(t\mathcal{L})$. From the view point of continuous measurement, this time evolution for the density operator corresponds to a situation in which we discard the measurement outcomes. (see Ref. [7] for a review).

The theory of the continuous quantum measurement has been studied from various standpoints. One is a mathematical approach [8–10]. There are also experimentally oriented pieces of work especially in quantum-optical systems [2, 14–17] and, more recently, in mesoscopic systems [18].

There are two distinct types of measurement processes: a jump-type and diffusive-type processes. In the jump-type measurement, discontinuous state changes occur at discrete times. A typical example is the photon-counting measurement, where the coupling between the photon field and the detector is adjusted so that the probability of more than one photon being detected during any infinitesimal time interval (i.e., the resolution time) is negligible. Therefore, the photon-counting measurement consists of two fundamental processes: no-count and one-count processes. The state change in the no-count process is, however, different from that of measurement-free evolution due to the back action of the measurement. On the other hand, in the diffusive-type measurement, the state change is continuous. Examples include balanced homodyne measurement [15] and continuous observations of the position of a particle [8]. In classical stochastic processes, the jump-type and diffusive-type continuous measurement correspond to the Poissonian and Wiener processes,

respectively [19, 20]

Mathematically, the hybrid type of continuous measurement with diffusive and jump outcomes is also possible. A general equation of the continuously observed system was derived under general semigroup assumptions by Barchielli *et al.* [10]. The simultaneous measurement of photon counting and homodyne detection analyzed in this paper is an example of the hybrid type continuous measurement. We note that a simultaneous measurement of photon counting and homodyne detection was discussed in Ref. [11, 12] in a different context. We also note that continuous measurements driven by Lévy processes are discussed in Ref. [13]

This paper is organized as follows. In Sec. II, we apply the general theory to the simultaneous measurement of photon counting and homodyne detection. We derive an analytic expression of the conditional wave function. In Sec. III, we examine probability laws of measurement records by deriving the probability distributions and the generating functional for measurement outcomes. In Sec. IV, we apply obtained general expressions to typical initial quantum states, namely, the coherent, number, thermal, and squeezed states. We also present the results of the Monte Carlo simulations for each of these initial states to illustrate how the hybrid-type measurement changes the average photon number. In Sec. V, we summarize the main results of this paper. In the Appendix, we show derivations of some formulae used in the main text.

II. HOMODYNE MEASUREMENT ACCOMPANIED BY PHOTON COUNTING

In this section, we consider a simultaneous measurement of photon counting and homodyne detection.

A. Setup of the system

The measurement scheme discussed in this section consists of photon counting and balanced homodyne detection. A single-mode photon field, described by the annihilation operator \hat{a} , is divided by a beam splitter into two, one of which is detected by a photodetector and the other is superimposed by a local oscillator with amplitude β and then measured by a balanced homodyne detector, as schematically illustrated in Fig. 1.

The measurement process during an infinitesimal time interval dt can be mathematically

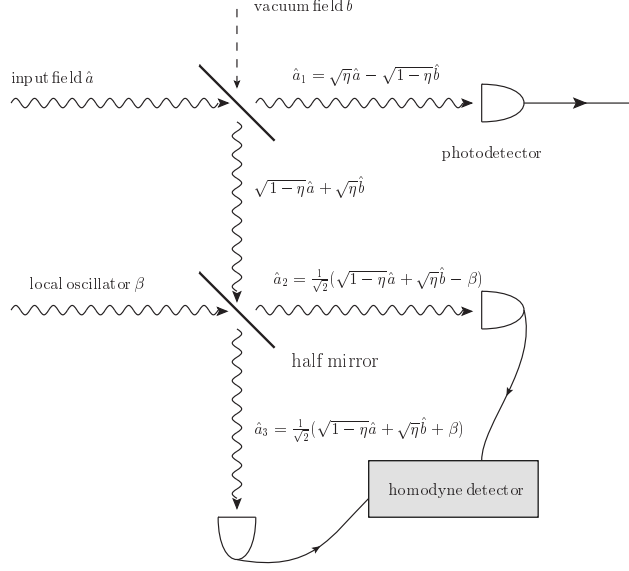


FIG. 1: Schematic illustration of the system. An input photon field is simultaneously measured by a photodetector and a balanced homodyne detector. See the text for details.

expressed by following measurement operators:

$$\hat{M}_{d\tilde{W}} = 1 - \left(i\omega + \frac{\gamma_1 + \gamma_2}{2} \right) \hat{n} dt + \sqrt{\gamma_2} \hat{a} d\tilde{W}, \quad (1)$$

$$\hat{M}_1 = \sqrt{\gamma_1} dt \hat{a}, \quad (2)$$

where $\gamma_1 > 0$ and $\gamma_2 > 0$ denote the coupling strength of the photodetector and that of the homodyne detector, respectively. The unitary part of the time evolution is given by $\hat{H} = \omega \hat{n}$, where ω is the detuning of the photon field with respect to the local oscillator β and $\hat{n} := \hat{a}^\dagger \hat{a}$ is the number operator of the photon field. The Planck constant \hbar is set to be 1 throughout this paper. $d\tilde{W}$ is the stochastic variable corresponding to the homodyne record with the expression [7]

$$d\tilde{W} = \langle \hat{a} + \hat{a}^\dagger \rangle dt + dW, \quad (3)$$

where $\langle \cdot \rangle := \text{tr}[\hat{\rho} \cdot]$ and dW is the Wiener increment which obeys the Itô rule $(dW)^2 = dt$ [13, 21, 22]. Equation (1) describes homodyne detection, while the measurement operator in Eq. (2) corresponds to the photodetection event. Actually, the vacuum field enters the mirror before the photodetector, but this effect can be neglected because it does not contribute to photodetector.

B. Wave function for the no-count process

Let the initial state vector of the system at $t = t_0$ be $|\psi_0\rangle$. We consider the time evolution of the wave function under the condition that the homodyne records are $\tilde{W}(\cdot)$ and that there is no photocount. It is described by the measurement operator in Eq. (1). Thus, the unnormalized wave function $|\tilde{\psi}(t)\rangle$ during the no-count process obeys

$$|\tilde{\psi}(t + dt)\rangle = \hat{M}_{d\tilde{W}}|\tilde{\psi}(t)\rangle = \left[1 - \left(i\omega + \frac{\Gamma}{2}\right)\hat{n}dt + \sqrt{\gamma_2}\hat{a}d\tilde{W}\right]|\tilde{\psi}(t)\rangle, \quad (4)$$

where $\Gamma := \gamma_1 + \gamma_2$ and the tilde over ψ indicates that the state vector is unnormalized. To solve this equation, let us introduce an interaction-picture wave function

$$|\tilde{\psi}_I(t)\rangle := \exp\left[(t - t_0)\left(i\omega + \frac{\Gamma}{2}\right)\hat{n}\right]|\tilde{\psi}(t)\rangle. \quad (5)$$

From Eq. (4), the time evolution of $|\tilde{\psi}_I(t)\rangle$ is given by

$$d|\tilde{\psi}_I(t)\rangle = \sqrt{\gamma_2}e^{-(i\omega + \frac{\Gamma}{2})(t-t_0)}\hat{a}d\tilde{W}_t|\tilde{\psi}_I(t)\rangle. \quad (6)$$

Therefore, we obtain

$$|\tilde{\psi}_I(t + dt)\rangle = \exp\left[\sqrt{\gamma_2}e^{-(i\omega + \frac{\Gamma}{2})(t-t_0)}\hat{a}d\tilde{W}_t - \frac{\gamma_2}{2}e^{-(2i\omega + \Gamma)(t-t_0)}\hat{a}^2dt\right]|\tilde{\psi}_I(t)\rangle. \quad (7)$$

Note that the second term in the exponent arises from the Itô rule $d\tilde{W}^2 = dt$. Since the two operators in the exponent of Eq. (7) commute with each other at any different time, Eq. (7) can be solved iteratively, giving

$$\begin{aligned} |\tilde{\psi}_I(t)\rangle &= \exp\left[\sqrt{\gamma_2}\int_{t_0}^t e^{-(i\omega + \frac{\Gamma}{2})(t'-t_0)}d\tilde{W}_{t'}\hat{a} - \frac{\gamma_2}{2}\int_{t_0}^t e^{-(2i\omega + \Gamma)(t'-t_0)}dt'\hat{a}^2\right]|\psi_0\rangle \\ &= \exp\left[\sqrt{\gamma_2}\int_{t_0}^t e^{-(i\omega + \frac{\Gamma}{2})(t'-t_0)}d\tilde{W}_{t'}\hat{a} - \frac{\gamma_2}{2}\frac{1 - e^{-(2i\omega + \Gamma)(t-t_0)}}{2i\omega + \Gamma}\hat{a}^2\right]|\psi_0\rangle. \end{aligned} \quad (8)$$

Going back to the original $|\tilde{\psi}(t)\rangle$, we obtain the unnormalized wave function for the no-count process:

$$\begin{aligned} |\tilde{\psi}(t)\rangle &= e^{-(i\omega + \frac{\Gamma}{2})(t-t_0)\hat{n}} \exp\left[\sqrt{\gamma_2}\int_{t_0}^t e^{-(i\omega + \frac{\Gamma}{2})(t'-t_0)}d\tilde{W}_{t'}\hat{a} - \frac{\gamma_2}{2}\frac{1 - e^{-(2i\omega + \Gamma)(t-t_0)}}{2i\omega + \Gamma}\hat{a}^2\right]|\psi_0\rangle \\ &=: \hat{N}(t, t_0; \tilde{W}(\cdot))|\psi_0\rangle. \end{aligned} \quad (9)$$

C. Wave function for the m -count process

We generalize the result obtained in the previous subsection to the conditional wave function under the m -count process. Let us assume that the initial condition of the wave function is $|\psi(t=0)\rangle = |\psi_0\rangle$. The conditional wave function under the condition that photocounts occur at times t_1, t_2, \dots, t_m during the time interval $(0, t)$ ($0 < t_1 < t_2 < \dots < t_m < t$) and that the homodyne records are $\tilde{W}(\cdot)$ is given by

$$|\tilde{\psi}(t)\rangle = (dt)^{m/2} |\tilde{\psi}(t; t_1, t_2, \dots, t_m; \tilde{W}(\cdot))\rangle, \quad (10)$$

where

$$\begin{aligned} & |\tilde{\psi}(t; t_1, t_2, \dots, t_m; \tilde{W}(\cdot))\rangle \\ &= \gamma_1^{m/2} \hat{N}(t, t_m; \tilde{W}(\cdot)) \hat{a} \hat{N}(t_m, t_{m-1}; \tilde{W}(\cdot)) \hat{a} \dots \hat{a} \hat{N}(t_2, t_1; \tilde{W}(\cdot)) \hat{a} \hat{N}(t_1, 0; \tilde{W}(\cdot)) |\psi_0\rangle. \end{aligned} \quad (11)$$

The product of the operators on the rhs of Eq. (11) can be simplified as

$$\begin{aligned} & \hat{N}(t, t_m; \tilde{W}(\cdot)) \hat{a} \hat{N}(t_m, t_{m-1}; \tilde{W}(\cdot)) \hat{a} \dots \hat{a} \hat{N}(t_2, t_1; \tilde{W}(\cdot)) \hat{a} \hat{N}(t_1, 0; \tilde{W}(\cdot)) \\ &= e^{-(i\omega + \frac{\Gamma}{2})(t_1 + t_2 + \dots + t_m)} e^{-(i\omega + \frac{\Gamma}{2})t\hat{n}} \hat{a}^m \exp[A_t \hat{a} + B(t) \hat{a}^2], \end{aligned} \quad (12)$$

where A_t and $B(t)$ are given by

$$A_t := \sqrt{\gamma_2} \int_0^t e^{-(i\omega + \frac{\Gamma}{2})t'} d\tilde{W}_{t'}, \quad (13)$$

$$B(t) := -\frac{\gamma_2}{2} \frac{1 - e^{-(2i\omega + \Gamma)t}}{2i\omega + \Gamma}. \quad (14)$$

Thus, the conditional m -count wave function is given by

$$\begin{aligned} & |\tilde{\psi}(t; t_1, t_2, \dots, t_m; \tilde{W}(\cdot))\rangle \\ &= \gamma_1^{m/2} e^{-(i\omega + \frac{\Gamma}{2})(t_1 + t_2 + \dots + t_m)} e^{-(i\omega + \frac{\Gamma}{2})t\hat{n}} \hat{a}^m \exp[A_t \hat{a} + B(t) \hat{a}^2] |\psi_0\rangle. \end{aligned} \quad (15)$$

Note that the time dependence on photocounts of this wave function arises, aside from the c-number factor $e^{-(i\omega + \frac{\Gamma}{2})(t_1 + t_2 + \dots + t_m)}$, only through the number of photocounts m .

D. Time Development of Expectation Values of Observables

For later use, we derive the equation for conditional time development of expectation value $\langle \hat{A} \rangle$ of an arbitrary operator \hat{A} .

For a one-count process, the density matrix $\hat{\rho}(t^+)$ immediately after the state $\hat{\rho}(t)$ at time t is given by $\hat{\rho}(t^+) = \hat{a}\hat{\rho}\hat{a}^\dagger/\langle\hat{n}\rangle$. Thus, the expectation value $\langle\hat{A}\rangle_+$ immediately after the photocount event is given by

$$\langle\hat{A}\rangle_+ = \frac{\langle\hat{a}^\dagger\hat{A}\hat{a}\rangle}{\langle\hat{n}\rangle}. \quad (16)$$

For a no-count process, the conditional evolution of the density matrix with homodyne record $d\tilde{W}$ is given by

$$\hat{\rho} + d\hat{\rho} = \frac{\hat{M}_{d\tilde{W}}\hat{\rho}\hat{M}_{d\tilde{W}}^\dagger}{\langle\hat{M}_{d\tilde{W}}^\dagger\hat{M}_{d\tilde{W}}\rangle}. \quad (17)$$

Substituting the expression of the measurement operator in Eq. (1), we obtain the equation for the differential of the expectation value $\langle\hat{A}\rangle$ in our system as follows:

$$\begin{aligned} d\langle\hat{A}\rangle &= \frac{\text{tr}[\hat{A}\hat{M}_{d\tilde{W}}\hat{\rho}\hat{M}_{d\tilde{W}}^\dagger]}{\text{tr}[\hat{M}_{d\tilde{W}}^\dagger\hat{M}_{d\tilde{W}}]} - \langle\hat{A}\rangle \\ &= \left[-i\omega\langle[\hat{A}, \hat{n}]\rangle - \frac{\gamma_1}{2}\langle\{\Delta\hat{A}, \Delta\hat{n}\}\rangle - \frac{\gamma_2}{2}\langle\{\hat{A}, \hat{n}\} - 2\hat{a}^\dagger\hat{A}\hat{a}\rangle \right] dt \\ &\quad + \sqrt{\gamma_2}\langle\Delta\hat{A}\Delta\hat{a} + \Delta\hat{a}^\dagger\Delta\hat{A}\rangle dW, \end{aligned} \quad (18)$$

where $\Delta\hat{A} := \hat{A} - \langle\hat{A}\rangle$.

Important examples of the expectation value $\langle\hat{A}\rangle$ are average photon number $\langle\hat{n}\rangle$ and quadrature amplitude $\langle\hat{a}\rangle$. By substituting \hat{n} and \hat{a} into \hat{A} , we obtain for the one-count process

$$\langle\hat{n}\rangle_+ = \frac{\langle\hat{n}^2\rangle - \langle\hat{n}\rangle}{\langle\hat{n}\rangle} = \langle\hat{n}\rangle - 1 + \frac{\langle[\Delta\hat{n}]^2\rangle}{\langle\hat{n}\rangle}, \quad (19)$$

$$\langle\hat{a}\rangle_+ = \frac{\langle\hat{a}^\dagger\hat{a}\hat{a}\rangle}{\langle\hat{n}\rangle} = \langle\hat{a}\rangle + \frac{\langle\Delta\hat{n}\Delta\hat{a}\rangle}{\langle\hat{n}\rangle}, \quad (20)$$

and for the no-count process

$$d\langle\hat{n}\rangle = -(\gamma_2\langle\hat{n}\rangle + \gamma_1\langle[\Delta\hat{n}]^2\rangle) dt + \sqrt{\gamma_2}\langle\Delta\hat{n}\Delta\hat{a} + \Delta\hat{a}^\dagger\Delta\hat{n}\rangle dW, \quad (21)$$

$$d\langle\hat{a}\rangle = -\left[\left(i\omega + \frac{\Gamma}{2}\right)\langle\hat{a}\rangle + \langle\Delta\hat{n}\Delta\hat{a}\rangle\right] dt + \sqrt{\gamma_2}\langle[\Delta\hat{a}]^2 + \Delta\hat{a}^\dagger\Delta\hat{a}\rangle dW. \quad (22)$$

III. PROBABILITY LAWS OF MEASUREMENT RECORDS

A. Probability density functions

In this section, we will derive general results on the probability distributions of homodyne and photocount records. From the general considerations on the measurement operators, the

joint probability density of homodyne records $\tilde{W}(\cdot)$ and photodetection times t_1, t_2, \dots, t_m is given by

$$dt_1 dt_2 \dots dt_m \times \mu_0(\tilde{W}(\cdot)) \times \langle \tilde{\psi}(t; t_1, t_2, \dots, t_m; \tilde{W}(\cdot)) | \tilde{\psi}(t; t_1, t_2, \dots, t_m; \tilde{W}(\cdot)) \rangle, \quad (23)$$

where μ_0 is the Wiener measure. The square of the norm of the wave function is evaluated to be

$$\begin{aligned} & \langle \tilde{\psi}(t; t_1, t_2, \dots, t_m; \tilde{W}(\cdot)) | \tilde{\psi}(t; t_1, t_2, \dots, t_m; \tilde{W}(\cdot)) \rangle \\ &= \gamma_1^m e^{-\Gamma(t_1+t_2+\dots+t_m)} \langle \psi_0 | \exp [A_t^* \hat{a}^\dagger + B^*(t)(\hat{a}^\dagger)^2] (\hat{a}^\dagger)^m e^{-\Gamma t \hat{n}} \hat{a}^m \exp [A_t \hat{a} + B(t)\hat{a}^2] | \psi_0 \rangle \\ &= \gamma_1^m e^{-\Gamma(t_1+t_2+\dots+t_m)} \langle \psi_0 | : e^{A_t \hat{a} + A_t^* \hat{a}^\dagger + B(t)\hat{a}^2 + B^*(t)(\hat{a}^\dagger)^2 - (1-e^{-\Gamma t})\hat{a}^\dagger \hat{a}} (\hat{a}^\dagger \hat{a})^m : | \psi_0 \rangle, \end{aligned} \quad (24)$$

where the symbol $: \dots :$ in Eq. (24) indicates normal ordering which places annihilation operators to the right of creation operators. In deriving the last equality in Eq. (24), we used the formula

$$e^{x\hat{n}} =: e^{\hat{a}^\dagger \hat{a}(e^x-1)} : \quad (25)$$

which is valid for an arbitrary c-number x . In the limit of $t \rightarrow \infty$, Eq. (24) reduces to

$$\gamma_1^m e^{-\Gamma(t_1+t_2+\dots+t_m)} \langle \psi_0 | : e^{A_\infty \hat{a} + A_\infty^* \hat{a}^\dagger + B(\infty)\hat{a}^2 + B^*(\infty)(\hat{a}^\dagger)^2 - \hat{a}^\dagger \hat{a}} (\hat{a}^\dagger \hat{a})^m : | \psi_0 \rangle, \quad (26)$$

where

$$A_\infty = \sqrt{\gamma_2} \int_0^\infty e^{-(i\omega + \frac{\Gamma}{2})t'} d\tilde{W}_{t'}, \quad (27)$$

$$B(\infty) = -\frac{\gamma_2}{2i\omega + \Gamma}. \quad (28)$$

The joint probability $\tilde{p}_m(t; \tilde{W}(\cdot))$ of m -photocounts being recorded during time interval $(0, t)$ and the homodyne records $\tilde{W}(\cdot)$ is obtained by integrating Eq. (24) with respect to t_1, t_2, \dots, t_m in the integration range $0 < t_1 < t_2 < \dots < t_m < t$. The relevant part of the integral is the exponential $e^{-\Gamma(t_1+t_2+\dots+t_m)}$ and evaluated to give

$$\begin{aligned} & \int_0^t dt_m \int_0^{t_m} dt_{m-1} \dots \int_0^{t_2} dt_1 e^{-\Gamma(t_1+t_2+\dots+t_m)} \\ &= \frac{1}{m!} \left(\frac{1 - e^{-\Gamma t}}{\Gamma} \right)^m. \end{aligned} \quad (29)$$

Thus, $\tilde{p}_m(t; \tilde{W}(\cdot))$ is given by

$$\begin{aligned} & \tilde{p}_m(t; \tilde{W}(\cdot)) \\ &= \langle \psi_0 | : \frac{1}{m!} \left(\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) \hat{a}^\dagger \hat{a} \right)^m e^{A_t \hat{a} + A_t^* \hat{a}^\dagger + B(t) \hat{a}^2 + B^*(t) (\hat{a}^\dagger)^2 - (1 - e^{-\Gamma t}) \hat{a}^\dagger \hat{a}} : | \psi_0 \rangle \end{aligned} \quad (30)$$

$$= \langle \psi_0 | e^{A_t^* \hat{a}^\dagger + B^*(t) (\hat{a}^\dagger)^2} \binom{\hat{n}}{m} e^{-\Gamma t (\hat{n} - m)} \left(\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) \right)^m e^{A_t \hat{a} + B(t) \hat{a}^2} | \psi_0 \rangle, \quad (31)$$

where in deriving the last equality we have used the formula

$$(\hat{a}^\dagger)^m e^{x \hat{n}} \hat{a}^m = \hat{n}(\hat{n} - 1) \cdots (\hat{n} - m + 1) e^{x(\hat{n} - m)}, \quad (32)$$

and defined the operator binomial coefficient by

$$\binom{\hat{n}}{m} := \frac{\hat{n}(\hat{n} - 1) \cdots (\hat{n} - m + 1)}{m!}. \quad (33)$$

In the limit $t \rightarrow \infty$, the joint probability in Eq. (30) reduces to

$$\langle \psi_0 | : \frac{1}{m!} \left(\frac{\gamma_1}{\Gamma} \hat{a}^\dagger \hat{a} \right)^m \exp [A_\infty \hat{a} + A_\infty^* \hat{a}^\dagger + B(\infty) \hat{a}^2 + B^*(\infty) (\hat{a}^\dagger)^2 - \hat{a}^\dagger \hat{a}] : | \psi_0 \rangle. \quad (34)$$

Note that Eq. (26) gives the total probability functional for the measurement outcomes and that its dependence on the homodyne records enters this formula only through A_∞ .

1. Marginal distribution of homodyne records

The marginal (unconditional) distribution of homodyne records $\tilde{p}(t; \tilde{W}(\cdot))$ can be obtained by taking the sum of $\tilde{p}_m(t; \tilde{W}(\cdot))$ over the number of photocounts m . This calculation precedes as follows:

$$\begin{aligned} & \tilde{p}(t; \tilde{W}(\cdot)) \\ &= \sum_{m=0}^{\infty} \tilde{p}_m(t; \tilde{W}(\cdot)) \\ &= \langle \psi_0 | : \frac{1}{m!} \left(\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) \hat{a}^\dagger \hat{a} \right)^m e^{A_t \hat{a} + A_t^* \hat{a}^\dagger + B(t) \hat{a}^2 + B^*(t) (\hat{a}^\dagger)^2 - (1 - e^{-\Gamma t}) \hat{a}^\dagger \hat{a}} : | \psi_0 \rangle \\ &= \langle \psi_0 | : e^{A_t \hat{a} + A_t^* \hat{a}^\dagger + B(t) \hat{a}^2 + B^*(t) (\hat{a}^\dagger)^2 - \frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) \hat{a}^\dagger \hat{a}} : | \psi_0 \rangle, \end{aligned} \quad (35)$$

which, in the limit of $t \rightarrow \infty$, reduces to

$$\tilde{p}(\infty; \tilde{W}(\cdot)) = \langle \psi_0 | : e^{A_\infty \hat{a} + A_\infty^* \hat{a}^\dagger + B(\infty) \hat{a}^2 + B^*(\infty) (\hat{a}^\dagger)^2 - \frac{\gamma_1}{\Gamma} \hat{a}^\dagger \hat{a}} : | \psi_0 \rangle. \quad (36)$$

This functional of $\tilde{W}(\cdot)$ gives the marginal distribution of the homodyne records.

2. Marginal distribution of photon-counting records

We can also derive the marginal photocount distribution of $p_m(t)$, which is the probability of m photocounts being registered in the time interval $(0, t)$ and can be derived by averaging $\tilde{p}_m(t; \tilde{W}(\cdot))$ over Wiener measure $\mu_0(\tilde{W}(\cdot))$. To evaluate the integral, we use the following general formula on the integration of a normally-ordered operator-valued function:

$$\int : f(\hat{a}, \hat{a}^\dagger; \omega) : \mu(d\omega) =: \int f(\hat{a}, \hat{a}^\dagger; \omega) \mu(d\omega) :, \quad (37)$$

where $f(\hat{a}, \hat{a}^\dagger; \omega)$ is a function of operators, \hat{a} and \hat{a}^\dagger , and of integration variable ω . The integration on the rhs of Eq. (37) can be carried out as if \hat{a} and \hat{a}^\dagger were c-numbers. By using the normally ordered expression of $\tilde{p}_m(t; \tilde{W}(\cdot))$ in Eq. (30) and the formula (37), the relevant part is the linear term on A_t :

$$\int \mu_0(\tilde{W}) e^{A_t \hat{a} + A_t^* \hat{a}^\dagger} = \int \mu_0(\tilde{W}) e^{\sqrt{\gamma_2} \int_0^t e^{-\frac{\Gamma}{2} t'} (e^{-i\omega t'} \hat{a} + e^{i\omega t'} \hat{a}^\dagger) d\tilde{W}_{t'}}. \quad (38)$$

By using the formula

$$\int \mu_0(\tilde{W}) e^{\int_0^t f(t') d\tilde{W}_{t'}} = e^{\frac{1}{2} \int_0^t f(t')^2 dt'}, \quad (39)$$

the integral on the rhs in Eq. (38) becomes

$$\exp \left[\frac{\gamma_2}{2} \left(\frac{1 - e^{-(2i\omega + \Gamma)t}}{2i\omega + \Gamma} \hat{a}^2 + \frac{1 - e^{-(-2i\omega + \Gamma)t}}{-2i\omega + \Gamma} (\hat{a}^\dagger)^2 \right) + \frac{\gamma_2}{\Gamma} (1 - e^{-\Gamma t}) \hat{a}^\dagger \hat{a} \right]. \quad (40)$$

Thus, the photocount probability distribution is given by

$$p_m(t) = \langle \psi_0 | : \frac{1}{m!} \left(\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) \hat{a}^\dagger \hat{a} \right)^m \exp \left[-\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) \hat{a}^\dagger \hat{a} \right] : | \psi_0 \rangle. \quad (41)$$

One may use Eq. (25) to rewrite Eq. (41) as follows:

$$p_m(t) = \langle \psi_0 | \binom{\hat{n}}{m} \left[\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) \right]^m \left[\frac{\gamma_1 e^{-\Gamma t} + \gamma_2}{\Gamma} \right]^{\hat{n}-m} | \psi_0 \rangle. \quad (42)$$

In the limit of $t \rightarrow \infty$, Eq. (42) reduces to

$$p_m(\infty) = \langle \psi_0 | : \frac{1}{m!} \left(\frac{\gamma_1}{\Gamma} \hat{a}^\dagger \hat{a} \right)^m \exp \left[\frac{\gamma_1}{\Gamma} \hat{a}^\dagger \hat{a} \right] : | \psi_0 \rangle \quad (43)$$

$$= \langle \psi_0 | \binom{\hat{n}}{m} \left(\frac{\gamma_1}{\Gamma} \right)^m \left(\frac{\gamma_2}{\Gamma} \right)^{\hat{n}-m} | \psi_0 \rangle. \quad (44)$$

B. Generating functional

In this subsection, we derive a general formula for the generating functional of measurement records $d\tilde{W}_t$ and dN_t , where dN_t is defined by

$$dN_t := \begin{cases} 1 & \text{if a photocount occurs;} \\ 0 & \text{otherwise.} \end{cases} \quad (45)$$

Instead of deriving the generating functional of $d\tilde{W}_t$ and dN_t , we discuss that of $dA_t = \sqrt{\gamma_2}e^{-(\frac{\Gamma}{2}+i\omega)t}d\tilde{W}_t$ and dN_t , which is defined as

$$M[\xi(\cdot), \xi^*(\cdot), \eta(\cdot)] = E \left[e^{\int_0^\infty \xi(t')dA_{t'} + \int_0^\infty \xi^*(t')dA_{t'}^* + \int_0^\infty \eta(t')dN_{t'}} \right], \quad (46)$$

where ξ and η are arbitrary functions of t . This functional contains all the information about the probability distribution of measurement records.

To calculate this generating functional, we note that the stochastic integral $\int_0^\infty \eta(t')dN_{t'}$ becomes $\sum_{k=1}^m \eta(t_k)$, if the photocounts occur at times $t_1 < t_2 < \dots < t_m$. Thus, the generating functional can be evaluated as

$$\begin{aligned} M[\xi(\cdot), \xi^*(\cdot), \eta(\cdot)] &= \sum_{m=0}^\infty \int_0^\infty dt_m \int_0^{t_m} dt_{m-1} \cdots \int_0^{t_2} dt_1 \int \mu_0(\tilde{W}(\cdot)) \\ &\times \langle \tilde{\psi}_\infty(t_1, t_2, \dots, t_m; \tilde{W}(\cdot)) | \tilde{\psi}_\infty(t_1, t_2, \dots, t_m; \tilde{W}(\cdot)) \rangle e^{\int_0^\infty \xi(t')dA_{t'} + \int_0^\infty \xi^*(t')dA_{t'}^* + \sum_{k=1}^m \eta(t_k)}. \end{aligned} \quad (47)$$

From the square norm of the wave function in Eq. (26), we obtain

$$\begin{aligned} &M[\xi(\cdot), \xi^*(\cdot), \eta(\cdot)] \\ &= \sum_{m=0}^\infty \int_0^\infty dt_m \int_0^{t_m} dt_{m-1} \cdots \int_0^{t_2} dt_1 \int \mu_0(\tilde{W}(\cdot)) \gamma_1^m e^{-\Gamma(t_1+t_2+\dots+t_m) + \sum_{k=1}^m \eta(t_k)} \\ &\times \langle \psi_0 | : e^{\int_0^\infty \xi(t')dA_{t'} + \int_0^\infty \xi^*(t')dA_{t'}^* + A_\infty \hat{a} + A_\infty^* \hat{a}^\dagger + B(\infty)\hat{a}^2 + B^*(\infty)(\hat{a}^\dagger)^2 - \hat{a}^\dagger \hat{a}} (\hat{a}^\dagger \hat{a})^m : | \psi_0 \rangle \\ &= \langle \psi_0 | : \sum_{m=0}^\infty \frac{1}{m!} \left(\gamma_1 \hat{a}^\dagger \hat{a} \int_0^\infty e^{-\Gamma t' + \eta(t')} dt' \right)^m \int \mu_0(\tilde{W}(\cdot)) \\ &\times e^{\sqrt{\gamma_2} \int_0^\infty d\tilde{W}_{t'} [e^{-(i\omega + \frac{\Gamma}{2})t'} (\xi(t') + \hat{a}) + e^{-(i\omega + \frac{\Gamma}{2})t'} (\xi^*(t') + \hat{a}^\dagger)] + B(\infty)\hat{a}^2 + B^*(\infty)(\hat{a}^\dagger)^2 - \hat{a}^\dagger \hat{a}} : | \psi_0 \rangle \\ &= \exp \left[\frac{\gamma_2}{2} \int_0^\infty |\xi(t')e^{-(i\omega + \frac{\Gamma}{2})t'} + \xi^*(t')e^{-(i\omega + \frac{\Gamma}{2})t'}|^2 dt' \right] \langle \psi_0 | : e^{\kappa \hat{a} + \kappa^* \hat{a}^\dagger + \nu \hat{a}^\dagger \hat{a}} : | \psi_0 \rangle, \end{aligned} \quad (48)$$

where

$$\kappa = \gamma_2 \int_0^\infty (e^{-(2i\omega + \Gamma)t'} \xi(t') + e^{-\Gamma t'} \xi^*(t')) dt', \quad (49)$$

$$\nu = \gamma_1 \int_0^\infty e^{-\Gamma t'} (e^{\eta(t')} - 1) dt'. \quad (50)$$

Equation (48) gives the general formula for the generating functional of measurement records.

The generating function $E[e^{\xi A_t + \xi^* A_t^* + \eta N_t}]$ with respect to output variables A_t and N_t can be derived by substituting

$$\xi(t') = \begin{cases} \xi & (0 < t' < t); \\ 0 & (\text{otherwise}), \end{cases} \quad (51)$$

$$\eta(t') = \begin{cases} \eta & (0 < t' < t); \\ 0 & (\text{otherwise}) \end{cases} \quad (52)$$

into Eq. (48). The result takes a form similar to Eq. (48):

$$\begin{aligned} & E[e^{\xi A_t + \xi^* A_t^* + \eta N_t}] \\ &= \exp \left[\frac{\gamma_2}{2} \left(\frac{1 - e^{-(2i\omega + \Gamma)t'}}{2i\omega + \Gamma} \xi^2 + \frac{1 - e^{-(2i\omega + \Gamma)t}}{-2i\omega + \Gamma} (\xi^*)^2 + 2 \frac{1 - e^{-\Gamma t}}{\Gamma} |\xi|^2 \right) \right] \\ & \times \langle \psi_0 | : \exp[\kappa_t \hat{a} + \kappa_t^* \hat{a}^\dagger + \nu_t \hat{a}^\dagger \hat{a}] : | \psi_0 \rangle, \end{aligned} \quad (53)$$

where

$$\kappa_t = \gamma_2 \left(\frac{1 - e^{-(2i\omega + \Gamma)t'}}{2i\omega + \Gamma} \xi + \frac{1 - e^{-\Gamma t}}{\Gamma} \xi^* \right), \quad (54)$$

$$\nu_t = \frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) (e^\eta - 1). \quad (55)$$

To gain the physical insights into the formulas (48) and (53), let us assume that \hat{a} and \hat{a}^\dagger were c-numbers. Then, the generating functional (48) would be that of independent stochastic processes, $d\tilde{W}_t$ and dN_t . Such a description is justified only when the initial state is a coherent state (see Sec. IV); otherwise, there will, in general, be correlations between these output records because of the noncommutativity of \hat{a} and \hat{a}^\dagger . In this sense, the measurement records give us information about the system's deviation from the coherent state as exemplified in the next section. From a stand point of measurement theory, the correlation between the two output records, which can be seen in the generating functional (48), reflect the backaction of one measurement channel on the other.

IV. APPLICATION TO TYPICAL INITIAL CONDITIONS

In this section we apply the general formulae obtained in the previous sections to typical quantum states: coherent, number, thermal, and squeezed states.

A. Coherent state

The coherent state $|\alpha\rangle$ is represented in the number-state basis as

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \quad (56)$$

where α is an arbitrary complex number. The coherent state is an eigenstate of the boson annihilation operator:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle. \quad (57)$$

The conditional wave function of the m -count process in Eq. (15) for an initial state $|\psi_0\rangle = |\alpha\rangle$ is given by

$$\begin{aligned} |\tilde{\psi}(t; t_1, t_2, \dots, t_m; \tilde{W}(\cdot))\rangle &= \gamma_1^{m/2} e^{-(i\omega + \frac{\Gamma}{2})(t_1 + t_2 + \dots + t_m)} \alpha^m e^{A_t \alpha + B(t) \alpha^2 - \frac{|\alpha|^2}{2}(1 - e^{-\Gamma t})} \\ &\times |e^{-(i\omega + \frac{\Gamma}{2})t\hat{n}} \alpha\rangle. \end{aligned} \quad (58)$$

In evaluating the m -count wave function, we have used Eq. (57) and the formula

$$e^{\lambda \hat{n}} |\alpha\rangle = e^{-\frac{|\alpha|^2}{2}(1 - |e^\lambda|^2)} |e^\lambda \alpha\rangle. \quad (59)$$

It follows from Eq. (58) that the normalized state vector does not depend on the measurement outcomes and is given by $|e^{-(i\omega + \frac{\Gamma}{2})t\hat{n}} \alpha\rangle$; the system develops deterministically.

The generating functional in Eq. (48) is evaluated as

$$\begin{aligned} M[\xi(\cdot), \xi^*(\cdot), \eta(\cdot)] \\ = \exp \left[\frac{\gamma_2}{2} \int_0^\infty |\xi(t') e^{-(i\omega + \frac{\Gamma}{2})t'} + \xi^*(t') e^{-(i\omega + \frac{\Gamma}{2})t'}|^2 dt' \right] \exp[\kappa \alpha + \kappa^* \alpha^* + \nu |\alpha|^2], \end{aligned} \quad (60)$$

where we used the formula

$$\langle \alpha | : f(\hat{a}, \hat{a}^\dagger) : | \alpha \rangle = f(\alpha, \alpha^*). \quad (61)$$

Note that the generating functional in Eq. (60) can be obtained by replacing \hat{a} and \hat{a}^\dagger by the corresponding c-numbers, α and α^* . From the definitions of κ and ν in Eqs. (49) and (50), we find from Eq. (60) that

$$\begin{aligned} M[\xi(\cdot), \xi^*(\cdot), \eta(\cdot)] &= M[\xi(\cdot), \xi^*(\cdot), \eta(\cdot) \equiv 0] \\ &\times M[\xi(\cdot) \equiv 0, \eta(\cdot)], \end{aligned} \quad (62)$$

which implies that homodyne records $\tilde{W}(\cdot)$ and photocount records are statistically independent.

From Eq. (53), the generating function of A_t is

$$\begin{aligned} E[e^{\xi A_t + \xi^* A_t^*}] &= \exp \left[\frac{\gamma_2}{2} \left(\frac{1 - e^{(2i\omega + \Gamma)t'}}{2i\omega + \Gamma} \xi^2 + \frac{1 - e^{-(2i\omega + \Gamma)t}}{-2i\omega + \Gamma} (\xi^*)^2 + 2 \frac{1 - e^{-\Gamma t}}{\Gamma} |\xi|^2 \right) \right. \\ &\quad \left. + \xi \gamma_2 \left(\alpha \frac{1 - e^{(2i\omega + \Gamma)t'}}{2i\omega + \Gamma} + \alpha^* \frac{1 - e^{-\Gamma t}}{\Gamma} \right) + \xi^* \gamma_2 \left(\alpha^* \frac{1 - e^{-(2i\omega + \Gamma)t}}{-2i\omega + \Gamma} + \alpha \frac{1 - e^{-\Gamma t}}{\Gamma} \right) \right], \end{aligned} \quad (63)$$

which implies that A_t is a complex Gaussian variable with its first and second moments given as follows:

$$E[A_t] = \gamma_2 \left(\alpha \frac{1 - e^{(2i\omega + \Gamma)t'}}{2i\omega + \Gamma} + \alpha^* \frac{1 - e^{-\Gamma t}}{\Gamma} \right), \quad (64)$$

$$E[(A_t - E[A_t])^2] = \frac{\gamma_2}{2} \frac{1 - e^{(2i\omega + \Gamma)t'}}{2i\omega + \Gamma}, \quad (65)$$

$$E[|A_t - E[A_t]|^2] = \gamma_2 \frac{1 - e^{-\Gamma t}}{\Gamma}. \quad (66)$$

From the normally-ordered representation (41), the marginal photocount distribution is evaluated as

$$p_m(t) = \frac{1}{m!} \left(\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) |\alpha|^2 \right)^m \exp \left[\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) |\alpha|^2 \right], \quad (67)$$

which is a Poisson distribution with its mean $\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) |\alpha|^2$.

B. Number state

The m -count wave function for an initial number state $|\psi_0\rangle = |n\rangle$ is evaluated as

$$\begin{aligned} &|\tilde{\psi}(t; t_1, t_2, \dots, t_m; \tilde{W}(\cdot))\rangle \\ &= \gamma_1^{m/2} e^{-(i\omega + \frac{\Gamma}{2})(t_1 + t_2 + \dots + t_m)} e^{-(i\omega + \frac{\Gamma}{2})\hat{n}t} \exp [A_t \hat{a} + B(t) \hat{a}^2] \hat{a}^m |n\rangle \\ &= \gamma_1^{m/2} e^{-(i\omega + \frac{\Gamma}{2})(t_1 + t_2 + \dots + t_m)} e^{-(i\omega + \frac{\Gamma}{2})\hat{n}t} \sum_{k=0}^{n-m} \sum_{l=0}^{[k/2]} \frac{A_t^{k-2l} B(t)^l}{(k-2l)! l!} \sqrt{\frac{n!}{(n-m-k)!}} |n-m-k\rangle \\ &= \gamma_1^{m/2} e^{-(i\omega + \frac{\Gamma}{2})(t_1 + t_2 + \dots + t_m)} \\ &\quad \times \sum_{k=0}^{n-m} \sum_{l=0}^{[k/2]} \frac{A_t^{k-2l} B(t)^l}{(k-2l)! l!} \sqrt{\frac{n!}{(n-m-k)!}} e^{-(i\omega + \frac{\Gamma}{2})(n-m-k)t} |n-m-k\rangle, \end{aligned} \quad (68)$$

where $[x]$ is the largest integer that does not exceed x . Note that $m \leq n$ since $|\tilde{\psi}(t; t_1, t_2, \dots, t_m; \tilde{W}(\cdot))\rangle$ vanishes for $m > n$.

The joint distribution function $\tilde{p}_m(t; \tilde{W}(\cdot))$ in Eq. (24) becomes

$$\begin{aligned} \tilde{p}_m(t; \tilde{W}(\cdot)) &= \left(\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) \right)^m \| e^{-(i\omega + \frac{\Gamma}{2})t\hat{n}} \exp[A_t \hat{a} + B(t) \hat{a}^2] \hat{a}^m |n\rangle \|^2 \\ &= \left(\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) \right)^m \sum_{k=0}^{n-m} \left| \sum_{l=0}^{[k/2]} \frac{A_t^{k-2l} B(t)^l}{(k-2l)! l!} \right|^2 \frac{n!}{(n-m-k)!} e^{-\Gamma t(n-m-k)}. \end{aligned} \quad (69)$$

In the limit of $t \rightarrow \infty$, Eq. (69) reduces to

$$\tilde{p}_m(\infty; \tilde{W}(\cdot)) = n! \left(\frac{\gamma_1}{\Gamma} \right)^m \left| \sum_{l=0}^{[(n-m)/2]} \frac{A_\infty^{k-2l} B(\infty)^l}{(n-m-2l)! l!} \right|^2. \quad (70)$$

The generating functional takes a simpler form. The calculation preceeds as follows: from

$$e^{\kappa \hat{a}} |n\rangle = \sum_{m=0}^n \frac{\kappa^m}{m!} \sqrt{\frac{n!}{(n-m)!}} |n-m\rangle, \quad (71)$$

we have

$$\begin{aligned} \langle n | : \exp[\kappa \hat{a} + \kappa^* \hat{a}^\dagger + \nu \hat{a}^\dagger \hat{a}] : | n \rangle &= (e^{\kappa \hat{a}} |n\rangle)^\dagger (1 + \nu)^{\hat{n}} e^{\kappa \hat{a}} |n\rangle \\ &= \sum_{m=0}^n \frac{n!}{(m!)^2 (n-m)!} |\kappa|^2 m (1 + \nu)^{n-m} \\ &= (1 + \nu)^n L_n \left(-\frac{|\kappa|^2}{1 + \nu} \right), \end{aligned} \quad (72)$$

where $L_n(x)$ is the Laguerre polynomial defined by

$$L_n(x) := \frac{e^x}{n!} \frac{d^n}{dx^n} (e^{-x} x^n) = \sum_{m=0}^n \frac{n!}{(m!)^2 (n-m)!} (-x)^m. \quad (73)$$

Thus, the generating functional and the generating function are given by

$$\begin{aligned} M[\xi(\cdot), \xi^*(\cdot), \eta(\cdot)] &= \exp \left[\frac{\gamma_2}{2} \int_0^\infty |\xi(t') e^{-(i\omega + \frac{\Gamma}{2})t'} + \xi^*(t') e^{-(i\omega + \frac{\Gamma}{2})t'}|^2 dt' \right] (1 + \nu)^n L_n \left(-\frac{|\kappa|^2}{1 + \nu} \right), \end{aligned} \quad (74)$$

$$\begin{aligned} E[e^{\xi A_t + \xi^* A_t^* + \eta N_t}] &= \exp \left[\frac{\gamma_2}{2} \left(\frac{1 - e^{-(2i\omega + \Gamma)t}}{2i\omega + \Gamma} \xi^2 + \frac{1 - e^{-(2i\omega + \Gamma)t}}{-2i\omega + \Gamma} (\xi^*)^2 + 2 \frac{1 - e^{-\Gamma t}}{\Gamma} |\xi|^2 \right) \right] \\ &\quad \times (1 + \nu_t)^n L_n \left(-\frac{|\kappa_t|^2}{1 + \nu_t} \right). \end{aligned} \quad (75)$$

Note that for the initial number state the generating function(al) cannot be factorized into a function of ξ and that of η , implying that there are correlations between these two measurement outputs.

From Eq. (42), the marginal counting distribution is

$$p_m(t) = \binom{n}{m} \left[\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) \right]^m \left[\frac{\gamma_1 e^{-\Gamma t} + \gamma_2}{\Gamma} \right]^{n-m}, \quad (76)$$

which is a binomial distribution.

The Monte Carlo simulation for the measurement process is done for the number-state initial condition. The simulation method is as follows: For each time step Δt , we first check if the photodetection occurs or not. If it does, the state vector or density operator evolves according to the jump operator in Eq. (2). If not, the state evolves according to the diffusive measurement operator in Eq. (1) corresponding to homodyne detection.

The results for an initial number state $|n\rangle$ with $n = 3$ are shown in Fig. 2 (a). In the no-count event, the expectation value of the photon number decreases in average according to Eq. (21), while there are local stochastic deviations which arise from the diffusive term $\sqrt{\gamma_2} \langle \Delta \hat{n} \Delta \hat{a} + \Delta \hat{a}^\dagger \Delta \hat{n} \rangle dW$ in Eq. (21).

C. Thermal state

Let us assume now that the initial state is a thermal state

$$\hat{\rho}_0 = \frac{e^{-\beta\omega\hat{n}}}{Z} = (1 - e^{-\beta\omega}) \sum_{n=0}^{\infty} e^{-\beta\omega n} |n\rangle \langle n|,$$

where

$$Z = \text{tr}[e^{-\beta\omega\hat{n}}] = \frac{1}{1 - e^{-\beta\omega}},$$

and β is the inverse temperature. Then, $\tilde{p}_m(t; \tilde{W}(\cdot))$ can be calculated by taking the ensemble average of the corresponding quantity for the initially number state over n :

$$\begin{aligned} \tilde{p}_m(t; \tilde{W}(\cdot)) &= \frac{1}{Z} \sum_{n=m}^{\infty} e^{-\beta\omega n} \left(\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) \right)^m \sum_{k=0}^{n-m} \left| \sum_{l=0}^{\lfloor k/2 \rfloor} \frac{A_t^{k-2l} B(t)^l}{(k-2l)! l!} \right|^2 \frac{n!}{(n-m-k)!} e^{-\Gamma t(n-m-k)} \\ &= \frac{e^{-\beta\omega m}}{Z} \left(\frac{\gamma_1}{\Gamma} (1 - e^{-\Gamma t}) \right)^m \sum_{k=0}^{\infty} \left| \sum_{l=0}^{\lfloor k/2 \rfloor} \frac{A_t^{k-2l} B(t)^l}{(k-2l)! l!} \right|^2 \frac{e^{-\beta\omega k} (k+m)!}{(1 - e^{-(\Gamma t + \beta\omega)})^{k+m+1}}, \end{aligned} \quad (77)$$

where in the last equality the following formula was used:

$$\sum_{n=0}^{\infty} \frac{(n+m)!}{(n-k)!} x^n = \frac{x^k (k+m)!}{(1-x)^{m+k+1}}. \quad (78)$$

The generating functional and the generating function are evaluated as follows:

$$\begin{aligned} M[\xi(\cdot), \xi^*(\cdot), \eta(\cdot)] &= \exp \left[\frac{\gamma_2}{2} \int_0^{\infty} |\xi(t') e^{-(i\omega + \frac{\Gamma}{2})t'} + \xi^*(t') e^{-(-i\omega + \frac{\Gamma}{2})t'}|^2 dt' \right] \sum_{n=0}^{\infty} \frac{e^{-\beta\omega n}}{Z} (1+\nu)^n L_n \left(-\frac{|\kappa|^2}{1+\nu} \right) \\ &= \frac{e^{\beta\omega} - 1}{e^{\beta\omega} - 1 - \nu} \exp \left[\frac{|\kappa|^2}{e^{\beta\omega} - 1 - \nu} + \frac{\gamma_2}{2} \int_0^{\infty} |\xi(t') e^{-(i\omega + \frac{\Gamma}{2})t'} + \xi^*(t') e^{-(-i\omega + \frac{\Gamma}{2})t'}|^2 dt' \right], \end{aligned} \quad (79)$$

$$\begin{aligned} E[e^{\xi A_t + \xi^* A_t^* + \eta N_t}] &= \frac{e^{\beta\omega} - 1}{e^{\beta\omega} - 1 - \nu_t} \exp \left[\frac{|\kappa_t|^2}{e^{\beta\omega} - 1 - \nu_t} \right. \\ &\quad \left. + \frac{\gamma_2}{2} \left(\frac{1 - e^{-(2i\omega + \Gamma)t'}}{2i\omega + \Gamma} \xi^2 + \frac{1 - e^{-(-2i\omega + \Gamma)t}}{-2i\omega + \Gamma} (\xi^*)^2 + 2 \frac{1 - e^{-\Gamma t}}{\Gamma} |\xi|^2 \right) \right]. \end{aligned} \quad (80)$$

In deriving the last equality in Eq. (79) we used the relation

$$\sum_{n=0}^{\infty} t^n L_n(x) = \frac{1}{1-t} \exp \left[-\frac{xt}{1-t} \right]. \quad (81)$$

Again, these characteristic functions are not separable with respect to ξ and η .

The marginal photocount distribution is

$$\begin{aligned} p_m(t) &= \sum_{n=0}^{\infty} \frac{e^{-\beta\omega n}}{Z} \binom{n}{m} p(t)^m (1-p(t))^{n-m} \\ &= \frac{e^{\beta\omega} - 1}{e^{\beta\omega} - 1 + p(t)} \left(\frac{p(t)}{e^{\beta\omega} - 1 + p(t)} \right)^m, \end{aligned} \quad (82)$$

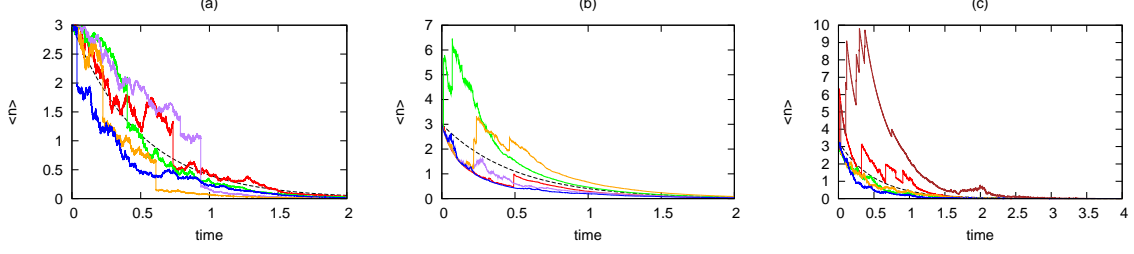


FIG. 2: (Color) Monte Carlo paths of $\langle \hat{n} \rangle$ starting from (a) number, (b) thermal and (c) squeezed states. In each figure, the dashed curve shows the ensemble average of the photon number which is given by $\langle \hat{n} \rangle_0 e^{-\Gamma t}$, where the subscript 0 indicates the average over the initial state. The parameters used are $\gamma_1 = \gamma_2 = 1$ and $\omega = 0$. For the number and thermal paths, the initial photon number is $\langle \hat{n} \rangle_0 = 3$. The parameters for the initial squeezed state are $r = 1.2$, $\alpha = 1$. For the case of (a), the change in $\langle \hat{n} \rangle$ upon photodetection is negative, while for (b) and (c), it is positive. This reflects the sub-Poissonian photon number distributions in (a) and the super-Poissonian distributions in (b) and (c). There also appears diffusive behavior in the no-count processes arising from homodyne detection in all of the three cases. The average behavior of damping (dashed curve) is consistent with sample paths.

where $p(t) := \frac{\gamma_1}{\Gamma}(1 - e^{-\Gamma t})$ and we have used (78) in deriving the last equality. We note that Eq. (82) is a geometric distribution.

The Monte Carlo paths of $\langle \hat{n} \rangle$ is shown in Fig. 2 (b). The average behavior showing an exponential damping in time is the same as the number state, while the change in the average photon number upon a photodetection is positive, reflecting the fact that the photon number distribution is super-Poissonian (see Eq. (21)) [16].

D. Squeezed state

Finally, we consider the case in which a squeezed state is taken as the initial condition:

$$|\alpha, r\rangle := D(\alpha)S(r)|0\rangle, \quad (83)$$

where

$$D(\alpha) := e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}}, \quad (84)$$

$$S(r) := e^{\frac{r}{2}(\hat{a}^2 - \hat{a}^{\dagger 2})}, \quad (85)$$

with α being an arbitrary complex number and r an arbitrary real number which is called a squeezing parameter.

To evaluate the generating function(al), we need to calculate

$$\langle \alpha, r | : \exp[\kappa \hat{a} + \kappa^* \hat{a}^\dagger + \nu \hat{a}^\dagger \hat{a}] : | \alpha, r \rangle. \quad (86)$$

This is done in Appendix A with the result

$$\begin{aligned} & \frac{1}{\sqrt{(1 + \frac{\nu}{2}(1 - e^{-2r})) (1 + \frac{\nu}{2}(1 - e^{2r}))}} \\ & \times \exp \left[-\frac{|\kappa|^2}{1 + \nu} + \frac{1}{\frac{2}{1+e^{-2r}} - \frac{\nu}{1+\nu}} \left(\frac{\text{Re}\kappa}{1 + \nu} + \frac{2\text{Re}\alpha}{1 + e^{-2r}} \right)^2 - \frac{2(\text{Re}\alpha)^2}{1 + e^{-2r}} \right. \\ & \left. + \frac{1}{\frac{2}{1+e^{2r}} - \frac{\nu}{1+\nu}} \left(\frac{-\text{Im}\kappa}{1 + \nu} + \frac{2\text{Im}\alpha}{1 + e^{2r}} \right)^2 - \frac{2(\text{Im}\alpha)^2}{1 + e^{2r}} \right]. \end{aligned} \quad (87)$$

Thus, the generating functional and the generating function are given by

$$\begin{aligned} & M[\xi(\cdot), \xi^*(\cdot), \eta(\cdot)] \\ & = \frac{1}{\sqrt{(1 + \frac{\nu}{2}(1 - e^{-2r})) (1 + \frac{\nu}{2}(1 - e^{2r}))}} \exp \left[\frac{\gamma_2}{2} \int_0^\infty |\xi(t') e^{-(i\omega + \frac{\Gamma}{2})t'} + \xi^*(t') e^{-(i\omega + \frac{\Gamma}{2})t'}|^2 dt' \right. \\ & - \frac{|\kappa|^2}{1 + \nu} + \frac{1}{\frac{2}{1+e^{-2r}} - \frac{\nu}{1+\nu}} \left(\frac{\text{Re}\kappa}{1 + \nu} + \frac{2\text{Re}\alpha}{1 + e^{-2r}} \right)^2 - \frac{2(\text{Re}\alpha)^2}{1 + e^{-2r}} \\ & \left. + \frac{1}{\frac{2}{1+e^{2r}} - \frac{\nu}{1+\nu}} \left(\frac{-\text{Im}\kappa}{1 + \nu} + \frac{2\text{Im}\alpha}{1 + e^{2r}} \right)^2 - \frac{2(\text{Im}\alpha)^2}{1 + e^{2r}} \right], \end{aligned} \quad (88)$$

$$\begin{aligned} & E[e^{\xi A_t + \xi^* A_t^* + \eta N_t}] \\ & = \frac{1}{\sqrt{(1 + \frac{\nu}{2}(1 - e^{-2r})) (1 + \frac{\nu}{2}(1 - e^{2r}))}} \exp \left[\frac{\gamma_2}{2} \left(\frac{1 - e^{-(2i\omega + \Gamma)t'}}{2i\omega + \Gamma} \xi^2 + \frac{1 - e^{-(2i\omega + \Gamma)t}}{-2i\omega + \Gamma} (\xi^*)^2 + 2 \frac{1 - e^{-\Gamma t}}{\Gamma} |\xi|^2 \right) \right. \\ & - \frac{|\kappa_t|^2}{1 + \nu_t} + \frac{1}{\frac{2}{1+e^{-2r}} - \frac{\nu_t}{1+\nu_t}} \left(\frac{\text{Re}\kappa_t}{1 + \nu_t} + \frac{2\text{Re}\alpha}{1 + e^{-2r}} \right)^2 - \frac{2(\text{Re}\alpha)^2}{1 + e^{-2r}} \\ & \left. + \frac{1}{\frac{2}{1+e^{2r}} - \frac{\nu_t}{1+\nu_t}} \left(\frac{-\text{Im}\kappa_t}{1 + \nu_t} + \frac{2\text{Im}\alpha}{1 + e^{2r}} \right)^2 - \frac{2(\text{Im}\alpha)^2}{1 + e^{2r}} \right]. \end{aligned} \quad (89)$$

We may use these results to describe the joint photocount and homodyne measurement process.

The Monte Carlo paths for initial squeezed state is shown in Fig. 2 (c). We take the initial parameters with $r = 1.2$ and $\alpha = 1$. The change in the average photon number upon photodetection is negative because of the super-Poissonian photon-number distribution.

V. CONCLUSION

We have introduced the measurement operators for the simultaneous measurement process of photoncounting and homodyne detection and derived the corresponding stochastic Schrödinger equation. These stochastic equations describe the time evolution of the quantum state under given measurement outcomes. The analytical expression of the conditional wave function is obtained and, using this expression, we have derived the probability density function and generating functional of measurement records as explicit functions of the initial state of the system. We have also derived the marginal distribution functions of two output channels and generating functionals. We have applied these general results to four typical initial conditions: coherent, number, thermal, and squeezed states. For each of these initial states, we have obtained analytical expressions of the generating functional of the measurement records. For the initial coherent state, the two output records for the photon counting and homodyne channels are mutually independent, while for the other states the two outputs are statistically dependent. In this sense, this measurement process extracts non-classicality of the system in the form of the correlation of two output channels. We have performed Monte Carlo simulations of the average photon number, which show properties of both photon counting and homodyne detection, implying the particle-wave duality of the photon field.

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Appendix A: Derivation of Eq. (87)

In this appendix, we derive Eq. (87) based on the Q-function technique.

The Q-function of state $\hat{\rho}$ is defined by [22]

$$Q(\beta, \beta^*) := \frac{1}{\pi} \langle \beta | \hat{\rho} | \beta \rangle. \quad (\text{A1})$$

This function is convenient for calculating the expectation values of anti-normally ordered operators. Let $f(\beta, \beta^*)$ be an arbitrary function of complex variables β, β^* with an expression

$$f(\beta, \beta^*) = \sum_{r,s} f_{rs} \beta^r (\beta^*)^s. \quad (\text{A2})$$

The anti-normally ordered operator of $f(\beta, \beta^*)$ is defined by

$$\mathcal{A}[f(\hat{a}, \hat{a}^\dagger)] := \sum_{r,s} f_{rs} \hat{a}^r (\hat{a}^\dagger)^s. \quad (\text{A3})$$

Then, from the overcompleteness relation of coherent states, we have [22]

$$\begin{aligned} \text{tr}(\hat{\rho} \mathcal{A}[f(\hat{a}, \hat{a}^\dagger)]) &= \sum_{r,s} f_{rs} \text{Tr} \left(\hat{\rho} \hat{a}^r \int \frac{d^2\beta}{\pi} |\beta\rangle \langle \beta| (\hat{a}^\dagger)^s \right) \\ &= \int \frac{d^2\beta}{\pi} \sum_{r,s} f_{rs} \beta^r (\beta^*)^s \text{Tr}(\hat{\rho} |\beta\rangle \langle \beta|) \\ &= \int d^2\beta f(\beta, \beta^*) Q(\beta, \beta^*). \end{aligned} \quad (\text{A4})$$

Equation (A4) implies that Q-function can be interpreted as a quasi-probability distribution function for anti-normally ordered operators. To exploit this property, we will calculate the Q-function of the squeezed state $|\alpha, r\rangle$ and the anti-normally ordered expression of the normally-ordered operator $:\exp[\kappa \hat{a} + \kappa^* \hat{a}^\dagger + \nu \hat{a}^\dagger \hat{a}] :$.

1. Q-function of the squeezed state

To evaluate the Q-function of the squeezed state $|\alpha, r\rangle$, we first prove following formula:

$$S(r) = \sqrt{\frac{1}{\cosh r}} e^{-\frac{(\hat{a}^\dagger)^2}{2} \tanh r} e^{-\hat{a}^\dagger \hat{a} \ln \cosh r} e^{\frac{\hat{a}^2}{2} \tanh r}. \quad (\text{A5})$$

To show this, we differentiate the rhs with respect to r :

$$\begin{aligned} &\frac{d}{dr}(\text{rhs}) \\ &= \sqrt{\frac{1}{\cosh r}} \left(-\frac{\tanh r}{2} e^{-\frac{(\hat{a}^\dagger)^2}{2} \tanh r} e^{-\hat{a}^\dagger \hat{a} \ln \cosh r} e^{\frac{\hat{a}^2}{2} \tanh r} - \frac{(\hat{a}^\dagger)^2}{2 \cosh^2 r} e^{-\frac{(\hat{a}^\dagger)^2}{2} \tanh r} e^{-\hat{a}^\dagger \hat{a} \ln \cosh r} e^{\frac{\hat{a}^2}{2} \tanh r} \right. \\ &\quad \left. + e^{-\frac{(\hat{a}^\dagger)^2}{2} \tanh r} (-\hat{a}^\dagger \hat{a} \tanh r) e^{-\hat{a}^\dagger \hat{a} \ln \cosh r} e^{\frac{\hat{a}^2}{2} \tanh r} + e^{-\frac{(\hat{a}^\dagger)^2}{2} \tanh r} e^{-\hat{a}^\dagger \hat{a} \ln \cosh r} \left(\frac{\hat{a}^2}{2 \cosh^2 r} \right) e^{\frac{\hat{a}^2}{2} \tanh r} \right) \\ &= \frac{\hat{a}^2 - (\hat{a}^\dagger)^2}{2} (\text{rhs}) \end{aligned} \quad (\text{A6})$$

In deriving the last equality, we used the following relations:

$$e^{-\gamma(\hat{a}^\dagger)^2} \hat{a} e^{\gamma(\hat{a}^\dagger)^2} = \hat{a} + 2\gamma\hat{a}^\dagger, \quad (\text{A7})$$

$$e^{-\gamma(\hat{a}^\dagger)^2} e^{-\lambda\hat{a}^\dagger\hat{a}} \hat{a} e^{\lambda\hat{a}^\dagger\hat{a}} e^{\gamma(\hat{a}^\dagger)^2} = e^\lambda(\hat{a} + 2\gamma\hat{a}^\dagger). \quad (\text{A8})$$

Equation (A6) shows that the rhs satisfies the same differential equation for the lhs. Since (lhs) = (rhs) = I when $r = 0$, Eq. (A5) holds for arbitrary r .

Using Eq. (A5) and

$$D(-\beta)D(\alpha) = e^{\frac{1}{2}(\alpha\beta^* - \alpha^*\beta)} D(\alpha - \beta),$$

we obtain

$$\begin{aligned} \langle\beta|D(\alpha)S(r)|0\rangle &= \langle 0|e^{\frac{1}{2}(\alpha\beta^* - \alpha^*\beta)} D(\alpha - \beta) \sqrt{\frac{1}{\cosh r}} e^{-\frac{(\hat{a}^\dagger)^2}{2} \tanh r} e^{-\hat{a}^\dagger \hat{a} \ln \cosh r} e^{\frac{\hat{a}^2}{2} \tanh r} |0\rangle \\ &= \frac{e^{\frac{1}{2}(\alpha\beta^* - \alpha^*\beta)}}{\sqrt{\cosh r}} \langle\beta - \alpha|e^{-\frac{(\hat{a}^\dagger)^2}{2} \tanh r}|0\rangle \\ &= \frac{e^{\frac{1}{2}(\alpha\beta^* - \alpha^*\beta)}}{\sqrt{\cosh r}} e^{-\frac{1}{2}(\beta^* - \alpha^*)^2 \tanh r - \frac{1}{2}|\beta - \alpha|^2}. \end{aligned}$$

Therefore, the Q-function for the squeezed state is given by

$$\begin{aligned} Q(\beta, \beta^*) &= \frac{1}{\pi} |\langle\beta|D(\alpha)S(r)|0\rangle|^2 \\ &= \frac{1}{\pi \cosh r} \exp \left[-|\beta - \alpha|^2 - \frac{\tanh r}{2} \{(\beta - \alpha)^2 + (\beta^* - \alpha^*)^2\} \right]. \quad (\text{A9}) \end{aligned}$$

2. Anti-normally ordered expression

We derive the anti-normally-ordered expression of the operator

$$: \exp[\kappa\hat{a} + \kappa^*\hat{a}^\dagger + \nu\hat{a}^\dagger\hat{a}] := e^{\kappa^*\hat{a}^\dagger} \left(: e^{\nu\hat{a}^\dagger\hat{a}} : \right) e^{\kappa\hat{a}}.$$

From

$$\mathcal{A}[(\hat{a}\hat{a}^\dagger)^m] = (\hat{n} + m)(\hat{n} + m - 1) \cdots (\hat{n} + 1), \quad (\text{A10})$$

$$\begin{aligned} \mathcal{A}[e^{x\hat{a}\hat{a}^\dagger}] &= \sum_{m=0}^{\infty} \frac{x^m}{m!} (\hat{n} + m)(\hat{n} + m - 1) \cdots (\hat{n} + 1) \\ &= (1 - x)^{-\hat{n}-1}, \quad (\text{A11}) \end{aligned}$$

we have

$$\begin{aligned}
: e^{\nu \hat{a}^\dagger \hat{a}} : &= (1 + \nu)^{\hat{a}^\dagger \hat{a}} \\
&= (1 + \nu)^{-1} \left(1 - \frac{\nu}{1 + \nu} \right)^{-\hat{a}^\dagger \hat{a} - 1} \\
&= (1 + \nu)^{-1} \mathcal{A}[e^{\frac{\nu}{1+\nu} \hat{a} \hat{a}^\dagger}].
\end{aligned} \tag{A12}$$

Using

$$e^{-\kappa \hat{a}} \hat{a}^\dagger e^{\kappa \hat{a}} = \hat{a}^\dagger - \kappa, \tag{A13}$$

$$e^{\kappa^* \hat{a}^\dagger} \hat{a} e^{-\kappa^* \hat{a}^\dagger} = \hat{a} - \kappa^*, \tag{A14}$$

we obtain

$$\begin{aligned}
&: \exp[\kappa \hat{a} + \kappa^* \hat{a}^\dagger + \nu \hat{a}^\dagger \hat{a}] : \\
&= (1 + \nu)^{-1} e^{\kappa^* \hat{a}^\dagger} \mathcal{A}[e^{\frac{\nu}{1+\nu} \hat{a} \hat{a}^\dagger}] e^{\kappa \hat{a}} \\
&= \frac{e^{-|\kappa|^2}}{1 + \nu} e^{\kappa \hat{a}} \mathcal{A}[e^{\frac{\nu}{1+\nu} (\hat{a} - \kappa^*)(\hat{a}^\dagger - \kappa)}] e^{\kappa^* \hat{a}^\dagger} \\
&= (1 + \nu)^{-1} \mathcal{A} \left[\exp \left[\frac{1}{1 + \nu} (-|\kappa|^2 + \kappa \hat{a} + \kappa^* \hat{a}^\dagger + \nu \hat{a} \hat{a}^\dagger) \right] \right] \\
&= \frac{e^{-\frac{|\kappa|^2}{1+\nu}}}{(1 + \nu)} \mathcal{A}[\exp[\kappa' \hat{a} + \kappa'^* \hat{a}^\dagger + \nu' \hat{a} \hat{a}^\dagger]],
\end{aligned} \tag{A15}$$

where

$$\begin{aligned}
\kappa' &:= \frac{\kappa}{1 + \nu}, \\
\nu' &:= \frac{\nu}{1 + \nu}.
\end{aligned}$$

3. Evaluation of Eq. (87)

We can then evaluate Eq. (87). The relevant part is

$$\begin{aligned}
& \langle \alpha, r | \mathcal{A} [\exp[\kappa' \hat{a} + \kappa'^* \hat{a}^\dagger + \nu' \hat{a} \hat{a}^\dagger]] | \alpha, r \rangle \\
&= \int d^2 \beta Q(\beta, \beta^*) \exp[\kappa' \beta + \kappa'^* \beta^* + \nu' |\beta|^2] \\
&= \int \frac{dx dy}{\pi \cosh r} \exp \left[-\frac{2}{1+e^{-2r}} (x - \text{Re} \alpha)^2 - \frac{2}{1+e^{2r}} (y - \text{Im} \alpha)^2 + \kappa' (x + iy) + \kappa'^* (x - iy) + \nu' (x^2 + y^2) \right] \\
&= \sqrt{\frac{1}{(1 - \frac{\nu'(1-e^{-2r})}{2})(1 - \frac{\nu'(1-e^{2r})}{2})}} \exp \left[\frac{1}{\frac{2}{1+e^{-2r}} - \nu'} \left(\text{Re} \kappa' + \frac{2 \text{Re} \alpha}{1+e^{-2r}} \right) - \frac{2(\text{Re} \alpha)^2}{1+e^{-2r}} \right. \\
&\quad \left. + \frac{1}{\frac{2}{1+e^{2r}} - \nu'} \left(-\text{Im} \kappa' + \frac{2 \text{Im} \alpha}{1+e^{2r}} \right) - \frac{2(\text{Im} \alpha)^2}{1+e^{2r}} \right]. \tag{A16}
\end{aligned}$$

Going back to original κ and ν , we obtain Eq. (87).

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